Work for Digital Signal Processing Supervision III

Please attempt all the questions below and submit your work by 6pm on the day before the supervision. Regardless of whether you submit the main body of the work in PDF or hand-written form, please include in it any code samples and their output, and also send email them to me as separate files so that I can run/unit test them. Please also include all your working and any additional thoughts with your work.

Questions

1. Examples sheet exercises 18–20.
2. Examination questions (all attached at the end of this document):
   (a) 2008 Paper 9 Question 11 – Parts (b) and (c).
   (b) 2014 Paper 8 Question 6 – all.
   (c) 2011 Paper 8 Question 6 – Parts (c) and (d).

2008 Paper 9 Question 11

Digital Signal Processing

(*a*) A radio system outputs signals with frequency components only in the range 2.5 MHz to 3.5 MHz. The analog-to-digital converter that you want to use to digitise such signals can be operated at sampling frequencies that are an integer multiple of 1 MHz. What is the *lowest* sampling frequency that you can use without destroying information through aliasing? [5 marks]

(*b*) Consider a digital filter with an impulse response for which the z-transform is

$$H(z) = \frac{(z+1)^2}{(z-0.7-0.7\mathrm{j})(z-0.7+0.7\mathrm{j})}$$

(*i*) Draw the location of zeros and poles of this function in relation to the complex unit circle. [2 marks]

(*ii*) If this filter is operated at a sampling frequency of 48 kHz, which (approximate) input frequency will experience the lowest attenuation? [2 marks]

(*iii*) Draw a direct form I block-diagram representation of this digital filter. [5 marks]

(*c*) Make the following statements correct by changing one word or number in each case. (Negating the sentence is not sufficient.)

(*i*) Statistical independence implies negative covariance.

(*ii*) Group 3 MH fax code uses a form of arithmetic coding.

(*iii*) Steven's law states that rational scales follow a logarithmic law.

(*iv*) The Karhunen–Loève transform is commonly approximated by the z-transform.

(*v*) 40 dB corresponds to an 80× increase in voltage.

(*vi*) The human ear has about 480 critical bands.

[6 marks]

2011 Paper 8 Question 6

Digital Signal Processing

(*a*) What can you say about the Fourier transform $X(f)$ if

(*i*) $x(t)$ is real; [2 marks]

(*ii*) $x(t) = -x(-t)$? [2 marks]

(*b*) Give the result of the Fourier transform $X(f) = \int_{-\infty}^{\infty} x(t)\,\mathrm{e}^{-2\pi \mathrm{j} f t}\,\mathrm{d}t$, using Dirac's delta where appropriate, of

(*i*) $x(t) = 1$; [1 mark]

(*ii*) $x(t) = \cos(2\pi t)$; [2 marks]

(*iii*) $x(t) = \mathrm{rect}(t)$; [2 marks]

(*iv*) $x(t) = [\frac{1}{2} + \frac{1}{2} \cdot \cos(2\pi t)] \cdot \mathrm{rect}(t)$. [3 marks]

(*c*) When is a random sequence $\{x_n\}$ called a "white noise" signal? [2 marks]

(*d*) Consider an n-dimensional random vector variable $\mathbf{X}$.

(*i*) How is its covariance matrix defined? [2 marks]

(*ii*) How can you change its representation without loss of information into a random vector of equal dimensionality in which all elements are mutually uncorrelated? [4 marks]

6 Digital Signal Processing (MGK)

(*a*) Consider a causal, order-2 digital filter with real-valued infinite impulse response sequence $h_0, h_1, h_2, \ldots$

(*i*) What is the z-transform $H(z)$ of this filter's impulse response? [2 marks]

(*ii*) Express $H(z)$ in terms of the locations c_1, c_2 of its two zeros and the locations d_1, d_2 of its two poles in $\mathbb{C}$. [4 marks]

(*iii*) Give a necessary condition for c_1, c_2, d_1, d_2 to ensure that $\{h_n\}$ has only real values. [4 marks]

(*iv*) If we operate that filter at sampling frequency f_s, what will its amplitude gain at frequency f be? [2 marks]

(*b*) A *notch filter* aims to suppress a single frequency f_c. One way of designing an order-2 notch filter, as in part (*a*), involves placing the zeros directly onto the unit circle, and the poles right next to them inside the unit circle, at distance $0 < \alpha < 1$ from 0:

$$c_1 = \mathrm{e}^{\mathrm{j}\omega}, \quad d_1 = \alpha \cdot c_1, \quad c_2 = \mathrm{e}^{-\mathrm{j}\omega}, \quad d_2 = \alpha \cdot c_2, \quad \text{with} \quad \omega = 2\pi f_\mathrm{c}/f_\mathrm{s}$$

(*i*) What is the z-transform of the impulse response of the resulting filter, written as a fraction of two polynomials of z^{-1}? [4 marks]

(*ii*) The *OxyMax* is a medical device designed in the United States. It processes a heart-beat signal with a sampling rate of $f_\mathrm{s} = 600$ Hz. It contains the following C function, which implements a notch filter, as in part (*b*)(*i*), to suppress in the input signal interference from the North American power grid at $f_\mathrm{c} = 60$ Hz:

```
double mains_notch(double sample) {
  static double x[4], y[4];
  static int n = 0;
  x[n&3] = sample;
  y[n&3] = sample + x[(n-1)&3] * b1 + x[(n-2)&3]
                  - y[(n-1)&3] * a1 - y[(n-2)&3] * a2;
  return y[n++&3];
}
```

The U.S. version initializes the constants used with `b1` $= 2\cos(\pi/5)$, `a1` $=$ `b1` $\times\, 0.9$ and `a2` $= 0.81$. What changed constant(s) will instead suppress the power-grid frequency at $f_\mathrm{c} = 50$ Hz for the European version? [4 marks]